PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST
UNIT 1

Section Two:
Calculator-assumed

Your Name: ______________________________

Your Teacher's Name: ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Mark	Max
8		8	15		10
9		11	16		15
10		8	17		7
11		12	18		7
12		8			
13		10			
14		4			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	11	11	100	100	67
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 11 Information Handbook 2021*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **(100 Marks)**

This section has **eleven** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8 {1.1.1, 1.1.2, 1.1.4} **(8 marks)**

A family of two adults and four children purchase 6 seats in a row for an Eagles football match. The seats are randomly allocated to members of the family.

(a) How many arranged ways can they be seated? (1 mark)

Solution
${}^{6}P_{6} = 6! = 720$
Specific behaviours
✓ correct number

(b) How many ways can they be seated if they must sit in the order of their ages? (1 mark)

Solution
No. Ways (Ascending Order) + No. Ways (Descending Order) = 1 + 1 = 2 ways
Specific behaviours
✓ correct number

(c) How many ways can they be seated if a child must be sitting on either end? (2 marks)

Solution
$4 \times 3 \times 4! = 288$
Specific behaviours
✓ correctly uses 2 children at ends ✓ correct number

(d) How many ways can they be seated if the two adults must sit together? (2 marks)

Solution
$5! \times 2! = 240$
Specific behaviours
✓ correctly uses 5 spaces for adults ✓ correct number

(e) How many ways can they be seated if the two youngest children do not sit together? (2 marks)

Solution
Total Ways – No. ways seated together $= 720 - 2! \times 5!$ $= 480$
Specific behaviours
✓ correctly uses complement of 2 children together ✓ correct number

Question 9 {1.1.5, 1.1.7} **(11 marks)**

Perth Modern School is sending a mixed volleyball team of 6 players to a State-wide competition. If there are 7 girls and 5 boys to choose from.

(a) How many different ways could the team be selected? (1 mark)

Solution
$^{12}C_6 = 924$
Specific behaviours
✓ correct number

(b) How many ways can the team containing 4 girls and 2 boys be selected? (2 marks)

Solution
$^{7}C_4 \times {}^{5}C_2 = 35 \times 10 = 350$
Specific behaviours
✓ Correctly uses separation of boys and girls ✓ correct number

(c) One of the girls, Mekides, is captain and must be chosen. How many teams of 6, containing Mekides, with a total of 4 girls and 2 boys can be selected? (2 marks)

Solution
$^{1}C_1 \times {}^{6}C_3 \times {}^{5}C_2 = 200$
Specific behaviours
✓ correctly uses selects captain with separation of boys and girls ✓ correct number

(d) How many teams of 6, containing at least 2 boys and at least 2 girls, can be selected? (3 marks)

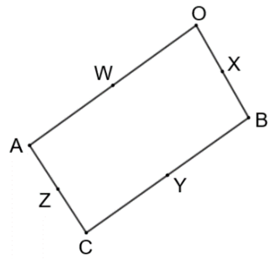
Solution
4G2B or 3G3B or 2G4B $= {}^{7}C_4 \times {}^{5}C_2 + {}^{7}C_3 \times {}^{5}C_3 + {}^{7}C_2 \times {}^{5}C_4$ $= 350 + 350 + 105$ $= 805$
Specific behaviours
✓ uses correct cases ✓ uses addition principle ✓ correct number

(e) A photograph is to be taken of a team of 3 boys and 3 girls selected from the total and then seated in a row for a photograph. How many possible photographs could be taken? (3 marks)

Solution
$^{7}C_3 \times {}^{5}C_3 \times 6! = 252000$
Specific behaviours
✓ correctly selects boys and girls ✓ correctly arranges students ✓ correct number

Question 10 {1.2.12, 1.3.17} **(8 marks)**

Let $OACB$ be a parallelogram and let W, X, Y and Z be midpoints of the sides as indicated in the diagram below.



Let $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{OB}$.

(a) Write expressions for $\overrightarrow{WX}$ and $\overrightarrow{XY}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. (3 marks)

Solution
$$\overrightarrow{WX} = \frac{1}{2}(\boldsymbol{b} - \boldsymbol{a})$$ Since $OABC$ is a parallelogram, $\overrightarrow{BC} = \overrightarrow{OA} = \boldsymbol{a}$. Hence $$\overrightarrow{XY} = \frac{1}{2}(\boldsymbol{b} + \boldsymbol{a})$$
Specific behaviours
✓ uses the fact that $OACB$ is a parallelogram ✓ correct expression for $\overrightarrow{WX}$ ✓ correct expression for $\overrightarrow{XY}$

(b) Use a vector method to prove that if $WXYZ$ is a rhombus, then $OACB$ is a rectangle. (5 marks)

Solution
Assume that $WXYZ$ is a rhombus. Then $$\left|\overrightarrow{WX}\right| = \left|\overrightarrow{XY}\right|$$ $$\left|\overrightarrow{WX}\right|^2 = \left|\overrightarrow{XY}\right|^2$$ $$\left|\frac{1}{2}(\boldsymbol{b} - \boldsymbol{a})\right|^2 = \left|\frac{1}{2}(\boldsymbol{b} + \boldsymbol{a})\right|^2$$ $$\frac{1}{4}(\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{b} - \boldsymbol{a}) = \frac{1}{4}(\boldsymbol{b} + \boldsymbol{a}) \cdot (\boldsymbol{b} + \boldsymbol{a})$$ $$\boldsymbol{b} \cdot \boldsymbol{b} - 2\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{a} = \boldsymbol{b} \cdot \boldsymbol{b} + 2\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{a}$$ $$-2\boldsymbol{a} \cdot \boldsymbol{b} = 2\boldsymbol{a} \cdot \boldsymbol{b}$$ $$\boldsymbol{a} \cdot \boldsymbol{b} = 0$$ From this it follows that $\boldsymbol{a} \perp \boldsymbol{b}$, meaning that sides $\overline{OA}$ and $\overline{OB}$ of $OACB$ are perpendicular. Since $OACB$ is a parallelogram with a right angle, it must be a rectangle.
Specific behaviours
✓ equates magnitudes of sides $\overrightarrow{WX}$ and $\overrightarrow{XY}$ ✓ substitutes squares of magnitudes with expressions involving scalar product ✓ expands scalar product expressions ✓ shows that $\boldsymbol{a} \cdot \boldsymbol{b} = 0$ ✓ concludes that $OACB$ is a rectangle

Question 11 {1.2.1, 1.2.2, 1.2.6, 1.2.7} **(12 marks)**

Relative to a fixed origin O, the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ are pointing due east and due north, respectively. The velocity of the particle, $\boldsymbol{v}$ in m/s, at time t in seconds after a given instant is

$$\boldsymbol{v} = (2 - 3t)\boldsymbol{i} + (3t - 6)\boldsymbol{j}.$$

(a) Find the speed of the particle when $t = 0$. (3 marks)

Solution
$t = 0, \boldsymbol{v} = 2\boldsymbol{i} - 6\boldsymbol{j}$ $\lvert\boldsymbol{v}\rvert = \sqrt{2^2 + (-6)^2}$ $= 2\sqrt{10}$ $= 6.32\ m/s$
Specific behaviours
✓ states velocity when $t = 0$ ✓ determines magnitude of velocity ✓ states magnitude with units

(b) Determine the bearing on which the particle is moving when $t = 4$. (3 marks)

Solution
$t = 4 \quad v = -10i + 6j$ $\theta = \tan^{-1}\left(\frac{6}{10}\right)$ $= 30.96°$ $\text{Bearing} = 270 + 30.96$ $= 300.96°$
Specific behaviours
✓ states velocity when $t = 4$ ✓ determines angle ✓ determines bearing

(c) Calculate the value of t when the particle is moving:

i. parallel to $\boldsymbol{i}$ (2 marks)

Solution
when moving parallel to i, the j component is zero $3t - 6 = 0$ $t = 2$
Specific behaviours
✓ equates $\boldsymbol{j}$ components ✓ correct answer

ii. parallel to $7\boldsymbol{i} - 5\boldsymbol{j}$. (4 marks)

Solution
$(2 - 3t)i + (3t - 6)j = \lambda(7i - 5j)$ $2 - 3t = 7\lambda \qquad 3t - 6 = -5\lambda$ $t = 5.33\ s$
Specific behaviours
✓ equates velocity vector to a scalar multiple of $7\boldsymbol{i} - 5\boldsymbol{j}$ ✓ equates $\boldsymbol{i}$ components ✓ equates $\boldsymbol{j}$ components ✓ correct answer

Question 12 {1.2.4} **(8 marks)**

Two forces F_1 and F_2 are acting on a particle so that the resultant of the two forces has a magnitude of 120 N acting on a bearing of 140°. F_1 acts due North and has a magnitude of 90 N.

(a) Represent F_2 as a vector in component form, to two decimal places. (4 marks)

Solution
$F_r = F_1 + F_2$ $(120\cos 50)i + (-120\sin 50)j = 90j + F_2$ $F_2 = (120\cos 50)i + (-120\sin 50 - 90)j$ $F_2 = 77.13i - 181.93j$
Specific behaviours
✓ gives resultant as the sum of F_1 and F_2 ✓ represents F_1 and F_r in component form ✓ gives correct $\boldsymbol{i}$ for F_2 ✓ gives correct $\boldsymbol{j}$ for F_2

(b) Calculate the magnitude of F_2 and the direction of this force, giving the answer as a three-digit bearing. (4 marks)

Solution
$F_2 = \sqrt{(77.13^2) + (181.93)^2}$ $= 198N$ $\theta = \tan^{-1}\left(\frac{181.93}{77.13}\right)$ $= 67°$ Bearing is $67 + 90 = 157'$
Specific behaviours
✓ sets up expression for magnitude of F_2 ✓ determines magnitude of F_2 ✓ finds the angle between F_2 and $\boldsymbol{i}$ ✓ determines bearing of F_2

Question 13 {1.2.10, 1.2.11, 1.2.13} **(10 marks)**

Relative to a fixed origin O, the points A and B have respective position vectors $3\boldsymbol{i} - \boldsymbol{j}$ and $2\boldsymbol{i} - 4\boldsymbol{j}$.

(a) Show that $\overrightarrow{OA}$ and $\overrightarrow{AB}$ are perpendicular. (3 marks)

Solution
For perpendicular vectors, $OA \cdot AB = 0$. $OA = 3i - j$ $AB = -OA + OB$ $= (-3i + j) + (2i - 4j)$ $= -i - 3j$ $OA \cdot AB = (3i - j) \cdot (-i - 3j)$ $= (-3) + 3$ $= 0$ $\therefore$ OA and AB are perpendicular.
Specific behaviours
✓ determines $\overrightarrow{AB}$ ✓ states that the dot product needs to be zero for perpendicular vectors ✓ shows that the dot product is zero

(b) A line is drawn through the points A and B. Point C lies on this line so that the area of the triangles OAB and OBC are equal. Determine the position vector of C. (2 marks)

Solution
For the area of the triangles to be equal, B must be the midpoint of the line drawn through A,B and C. $\left(\frac{3+x}{2}, \frac{-1+y}{2}\right) = (2,-4)$ hence $x = 1, y = -7$ $\Rightarrow$ point C is at $(1,-7)$ $OC = i - 7j$
Specific behaviours
✓ recognizes that B is the midpoint ✓ determines position vector of C

(c) Use a vector method, find the shortest distance from B to the line drawn from the origin through point C.

(5 marks)

Solution
Let u ⇒ the vector projection of OB in the direction of the line through point C w ⇒ vertical component of the vector OB perpendicular to u $u = \dfrac{OB \cdot OC}{OC \cdot OC} OC$ $= \dfrac{2 + 28}{1 + 49}(i - 7j)$ $= \dfrac{3}{5}(i - 7j)$ $w = -OB + u$ $= (-2i + 4j) + \dfrac{3}{5}(i - 7j)$ $= -\dfrac{7}{5}i - \dfrac{1}{5}j$ $\lvert w \rvert = \sqrt{\left(\dfrac{7}{5}\right) + \left(\dfrac{1}{5}\right)^2}$ $= \sqrt{2}$ $= 1.41$ units
Specific behaviours
✓ sets up to solve for the vector projection ✓ gives vector projection of $\overrightarrow{OB}$ in the direction of the line through C ✓ uses vector sum to solve for vertical component of $\overrightarrow{OB}$ perpendicular to u ✓ gives $\boldsymbol{w}$ in component form ✓ correct value for magnitude of $\boldsymbol{w}$

Question 14 {1.1.6} **(4 marks)**

A bag contains four blue, six red and seven green balls. How many balls must be chosen at random to guarantee that you will obtain:

(a) at least two balls of the same colour? (1 mark)

Solution
3 "boxes" with 1 in each "box" plus one extra → 4 balls chosen
Specific behaviours
✓ correct number

(b) at least three balls of the same colour? (1 mark)

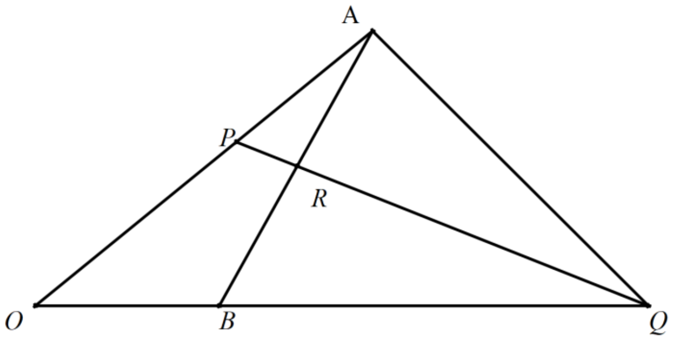
Solution
3 "boxes" with 2 in each "box" plus one extra → 7 balls chosen
Specific behaviours
✓ correct number

(c) at least six balls of the same colour? (2 marks)

Solution
3 "boxes" with 4 in each "box" plus 2 "boxes" with 1 in each "box" one extra = 4 + 4 + 4 + 1 + 1 +1 = 15 balls chosen
Specific behaviours
✓ allows for only having 4 blue balls ✓ correct number

Question 15 {1.2.2 – 1.2.12} **(10 marks)**

Consider the figure below showing triangle OAQ which is not drawn to scale. The point P lies on OAsuch that $OP: OA = 3:5$ and the point B lies on OQ such that $OB: OQ = 1:3$



Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OQ} = \boldsymbol{b}$

(a) Given $\overrightarrow{AR} = h\overrightarrow{AB}$ where h is a scalar, show that $\overrightarrow{OR} = (1-h)\boldsymbol{a} + \frac{1}{3}h\boldsymbol{b}$ (3 marks)

Solution

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -\underset{\sim}{a} + \tfrac{1}{3}\underset{\sim}{b}$$
$$\overrightarrow{AR} = h\,\overrightarrow{AB}$$
$$= h\left(-\underset{\sim}{a} + \tfrac{1}{3}\underset{\sim}{b}\right)$$
$$\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR}$$
$$= \underset{\sim}{a} + h\left(-\underset{\sim}{a} + \tfrac{1}{3}\underset{\sim}{b}\right)$$
$$= \underset{\sim}{a} - h\underset{\sim}{a} + \tfrac{1}{3}h\underset{\sim}{b}$$
$$= (1-h)\underset{\sim}{a} + \tfrac{1}{3}h\underset{\sim}{b}$$

Specific behaviours

✓ obtains an equation for $\overrightarrow{AB}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$
✓ obtains an equation for $\overrightarrow{OR}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and h
✓ simplifies to obtain $\overrightarrow{OR}$ as required

(b) Given $\overrightarrow{PR} = k\overrightarrow{PQ}$ where k is a scalar, show that $\overrightarrow{OR} = \frac{3}{5}(1-k)\boldsymbol{a} + k\boldsymbol{b}$ (3 marks)

Solution

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$$
$$= -\tfrac{3}{5}\underset{\sim}{a} + \underset{\sim}{b}$$
$$\overrightarrow{PR} = k\,\overrightarrow{PQ}$$
$$= k\left(-\tfrac{3}{5}\underset{\sim}{a} + \underset{\sim}{b}\right)$$
$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$$
$$= \tfrac{3}{5}\underset{\sim}{a} + k\left(-\tfrac{3}{5}\underset{\sim}{a} + \underset{\sim}{b}\right)$$
$$= \tfrac{3}{5}\underset{\sim}{a} - \tfrac{3}{5}k\underset{\sim}{a} + k\underset{\sim}{b}$$
$$= \tfrac{3}{5}(1-k)\underset{\sim}{a} + k\underset{\sim}{b}$$

Specific behaviours

✓ obtains an equation for $\overrightarrow{PQ}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$
✓ obtains an equation for $\overrightarrow{PR}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and k
✓ simplifies to obtain $\overrightarrow{OR}$ as required

Question 15 continued

(c) Determine the value of k and the value of h (3 marks)

Solution
$(1-h)\underset{\sim}{a} + \frac{1}{3}h\underset{\sim}{b} = \frac{3}{5}(1-k)\underset{\sim}{a} + k\underset{\sim}{b}$ $1-h = \frac{3}{5}(1-k) \Rightarrow 5-5h = 3-3k$ $\frac{1}{3}h = k \Rightarrow h = 3k$ $\therefore 5-5(3k) = 3-3k$ $5-15k = 3-3k$ $12k = 2$ $k = \frac{1}{6} \Rightarrow h = \frac{1}{2}$
Specific behaviours
✓ equates the two expressions for $\overrightarrow{OR}$ ✓ obtains correct value for k   ✓ obtains correct value for h

(d) Determine the ratio of $PR: PQ$ (1 mark)

Solution
$\overrightarrow{PR} = k\overrightarrow{PQ}$ $= \frac{1}{6}\overrightarrow{PQ}$ $\therefore PR : PQ = 1:6$
Specific behaviours
✓ States the correct ratio

Question 16 {1.2.1 – 1.2.5} **(15 marks)**

(a) Points A, B and C have position vectors $\overrightarrow{OA} = 5\boldsymbol{i} - 7\boldsymbol{j}$, $\overrightarrow{OB} = 8\boldsymbol{i} - 15\boldsymbol{j}$ and $\overrightarrow{OC} = -4\boldsymbol{i} + k\boldsymbol{j}$. Find the **exact** value(s) of k if

i. A, B and C are collinear. (3 marks)

Solution

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 3\boldsymbol{i} - 8\boldsymbol{j}$$

and

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = -12\boldsymbol{i} + (k+15)\boldsymbol{j}$$

Assume that A, B and C are collinear. Then $\overrightarrow{AB} = t\overrightarrow{BC}$ for some $t \in \mathbb{R}$, and so

$$(3\boldsymbol{i} - 8\boldsymbol{j}) = t(-12\boldsymbol{i} + (k+15)\boldsymbol{j})$$

Hence $3 = -12t$, and $-8 = t(k+15)$.
Solving the first equation gives $t = -\frac{1}{4}$, and then the second equation yields $k = 17$.

Specific behaviours

✓ calculates $\overrightarrow{AB}$ and $\overrightarrow{BC}$
✓ writes that $(3\boldsymbol{i} - 8\boldsymbol{j}) = t(-12\boldsymbol{i} + (k+15)\boldsymbol{j})$ and solves for t
✓ obtains correct value of k

ii. the distance from A to B is half the distance from B to C. (3 marks)

Solution

Assume that $|\overrightarrow{AB}| = \frac{1}{2}|\overrightarrow{BC}|$. Then

$$\sqrt{3^2 + 8^2} = \frac{1}{2}\sqrt{12^2 + (k+15)^2}$$

Solving for k gives

$$k = -15 \pm 2\sqrt{37}$$

Specific behaviours

✓ writes that $|\overrightarrow{AB}| = \frac{1}{2}|\overrightarrow{BC}|$
✓ sets up quadratic equation in k
✓ solves to get 2 correct exact values for k

(b) Consider the 2-dimensional vectors $\boldsymbol{a} = 4\boldsymbol{i} - 2\boldsymbol{j}$, $\boldsymbol{b} = 3\boldsymbol{i} + 6\boldsymbol{j}$ and $\boldsymbol{c} = x\boldsymbol{i} + y\boldsymbol{j}$. Explain why there can be no solutions for x and y satisfying both $\boldsymbol{a} \cdot \boldsymbol{c} = 0$ and $\boldsymbol{b} \cdot \boldsymbol{c} = 0$.
(3 marks)

Solution

If $\boldsymbol{a} \cdot \boldsymbol{c} = 0$ and $\boldsymbol{b} \cdot \boldsymbol{c} = 0$, then $\boldsymbol{c}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$, and it follows that $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel.

But $\boldsymbol{a} \cdot \boldsymbol{b} = 4 \times 3 + (-2) \times 6 = 0$, implying that $\boldsymbol{a} \perp \boldsymbol{b}$. Hence there can be no solutions for x and y satisfying $\boldsymbol{a} \cdot \boldsymbol{c} = 0$ and $\boldsymbol{b} \cdot \boldsymbol{c} = 0$.

Specific behaviours

✓ argues that if $\boldsymbol{a} \cdot \boldsymbol{c} = 0$ and $\boldsymbol{b} \cdot \boldsymbol{c} = 0$ then $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel
✓ evaluates $\boldsymbol{a} \cdot \boldsymbol{b}$ to show that $\boldsymbol{a} \perp \boldsymbol{b}$
✓ concludes that there can be no solutions for x and y

Question 16 continued

(c) XYZ is an equilateral triangle. If $\overrightarrow{XY} \cdot \overrightarrow{XZ} = 8$, determine

i. $\overrightarrow{ZY} \cdot \overrightarrow{ZX}$ (2 marks)

Solution
Since XYZ is equilateral, $\lvert\overrightarrow{XY}\rvert = \lvert\overrightarrow{ZY}\rvert = \lvert\overrightarrow{XZ}\rvert$ It follows from above that $\overrightarrow{ZY} \cdot \overrightarrow{ZX} = \lvert\overrightarrow{ZY}\rvert\lvert\overrightarrow{ZX}\rvert \cos 60$ $= \lvert\overrightarrow{XY}\rvert\lvert\overrightarrow{XZ}\rvert \cos 60$ $= \overrightarrow{XY} \cdot \overrightarrow{XZ}$ $= 8$
Specific behaviours
✓ argues $\overrightarrow{ZY} \cdot \overrightarrow{ZX} = \overrightarrow{XY} \cdot \overrightarrow{XZ}$ since XYZ is equilateral ✓ obtains correct value

ii. $\overrightarrow{XY} \cdot \overrightarrow{YZ}$ (2 marks)

Solution
The angle between $\overrightarrow{XY}$ and $\overrightarrow{YZ}$ is $180° - 60° = 120°$. Hence $\overrightarrow{XY} \cdot \overrightarrow{YZ} = \lvert\overrightarrow{XY}\rvert\lvert\overrightarrow{YZ}\rvert \cos 120$ $= -\lvert\overrightarrow{XY}\rvert\lvert\overrightarrow{YZ}\rvert \cos 60$ $= -8$
Specific behaviours
✓ identifies that the angle between $\overrightarrow{XY}$ and $\overrightarrow{YZ}$ is $120°$. ✓ obtains correct value

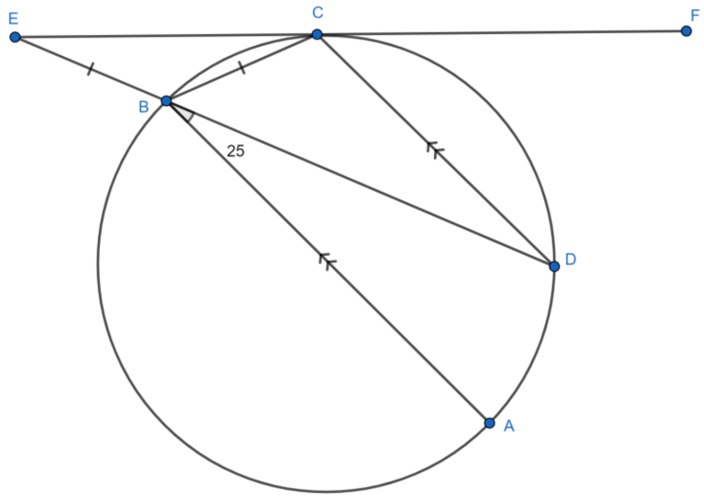
iii. $\lvert\overrightarrow{XY}\rvert$ (2 marks)

Solution
Since $\lvert\overrightarrow{XY}\rvert = \lvert\overrightarrow{XZ}\rvert$, $\overrightarrow{XY} \cdot \overrightarrow{XZ} = \lvert\overrightarrow{XY}\rvert\lvert\overrightarrow{XZ}\rvert \cos 60$ $= \lvert\overrightarrow{XY}\rvert^2 \cos 60$ $= \lvert\overrightarrow{XY}\rvert^2 \times \frac{1}{2}$ $= 8$ Hence $\lvert\overrightarrow{XY}\rvert^2 = 16$ and $\lvert\overrightarrow{XY}\rvert = 4$.
Specific behaviours
✓ evaluates $\overrightarrow{XY} \cdot \overrightarrow{XZ}$ using $\cos 60 = \frac{1}{2}$ ✓ obtains correct value

latest

Question 17 {1.3.11 – 1.3.15} **(7 marks)**

Consider the diagram below (not to scale) showing a circle with diameter AB. Points C and D also lie on the circle and the line EF is a tangent to the circle at point C. AB is parallel to CD and BC = BE. EBD is a secant to the circle and angle ABD is 25^0



(a) Find the size of $\angle BEC$ (3 marks)

Solution
$\angle ABD = \angle BDC = 25^0$ (Alternate angles) $\angle BCE = \angle BDC = 25^0$ (Angle in the alternate segment) $\angle BEC = \angle BCE = 25^0$ (Base angles of an isosceles triangle)
Specific Behaviours
Identifies an alternative angle to $\angle ABD$ Uses the alternate segment theorem States correct $\angle BEC$

(b) Find the size of $\angle ACF$ (3 marks)

Solution
$\angle ACF = \angle CBA$ (Angle in the alternate segment) $\angle EBC = 130^0$ (Triangle interior angle sum) $\angle CBD = 50^0$ (Angles in a straight line) $\angle ACF = \angle ABC = 75^0$ (Angle in the alternate segment)
Specific Behaviours
Identifies an alternative angle to $\angle ACF$ Uses triangle interior angle sum States correct $\angle ACF$

(c) Using relevant circle theorems, find the exact length of the base of ΔBCE given that $EB = 2\ cm$ and $EB: BD = 1: 2.5$ (1 mark)

Solution
$EC^2 = EB \times ED$ $EC^2 = 2 \times 7$ $EC = \pm\sqrt{14}$ The base of the triangle is $\sqrt{14}$ cm
Specific Behaviours
Uses power of a point theorem to obtain correct exact length

Question 18 {1.2.14} **(7 marks)**

At 10 am, electronic Hot Wheels A and B are launched from their locations at $(7i + 3j)\ m$ and $(-i - 0.8j)m$ with constant velocities $(-i + 0.25j)\ m/s$ and $(i + 1.2j)m/s$ respectively.

(a) Write vector expressions for $\boldsymbol{r_A(t)}$, the position vector of A at t second, and $\boldsymbol{r_B(t)}$, the position vector of B at t second. (2 marks)

(b) Write a vector expression for ${}^{A}\boldsymbol{r_B}(\boldsymbol{t})$, the relative position of A with respect to B at t second. (1 mark)

(c) Hence, show that the cars will collide if these velocities are maintained. State when and where the collision occurs. (4 marks)

Solution

The position vectors of both toy cars at time t is given by:

$$OA(t) = (7i + 3j) + (-i + 0.25j)t$$
$$= (7 - t)i + (3 + 0.25t)j$$
$$OB(t) = (-i - 0.8j) + (i + 1.2j)t$$
$$= (-1 + t)i + (-0.8 + 1.2t)j$$

$${}_a r_b\,(t) = OA - OB$$
$$= (8 - 2t)i + (3.8 - 0.95t)j$$

At the point of collision, the relative displacement is equal to the zero vector.

Solving for t for any of the components:

$$0 = 8 - 2t$$
$$t = 4$$

Both objects will collide after 4 seconds.

$$OA(4) = (7 - 4)i + (3 + (0.25 \times 4))j$$
$$= 3i + 4j$$
$$OB(4) = (-1 + 4)i + (-0.8 + (1.2 \times 4))j$$
$$= 3i + 4j$$

Both objects are at the same position at 4 seconds.

$\therefore$ Both toys will collide after 4 seconds at the point $(3i + 4j)$.

Specific behaviours

Part (a)
✓ gives equation for the position vector of $\overrightarrow{OA}$ as a function of time t.
✓ gives equation for the position vector of $\overrightarrow{OB}$ as a function of time t.

Part (b)
✓ finds the relative position of A with respect to B

Part (c)
✓ equates the relative displacement to 0.
✓ solves for t.
✓ shows that the position vectors of both cars are the same after 4 seconds.
✓ states correct position vector.

Additional working space

Question number: _______

Additional working space

Question number: ________

Additional working space

Question number: ________

Additional working space

Question number: ________

Additional working space

Question number: ________